

Experiment:

Claim: 20 poker chips.
10 red chip, rest blue
15 trials

n= 15
p= 0.5

RED	BLUE
11	

$$\hat{p} = 0.133$$

Given the info above, what's the chance of getting our experimental result or something more extreme?

$P(\hat{p} < 0.133) = \text{normalcdf}(-0.99, 0.133, 0.5, \sqrt{\frac{0.5(0.5)}{15}})$ Do you think the claim is true? Why?

$$= 0.0022$$

Claim: 20 poker chips.
 5 red chip, rest blue
 15 trials

$n = 15$
 $p = 0.25$

RED	BLUE
11	
	

$$\hat{p} = \frac{2}{15} = 0.133$$

Given the info above, what's the chance of getting our experimental result or something more extreme?

$P(\hat{p} < 0.133) = \text{normcdf}(-\infty, 0.133, 0.25, \sqrt{\frac{(0.25)(0.75)}{15}})$ Do you think the claim is true? Why?

$$0.25, \sqrt{\frac{(0.25)(0.75)}{15}} = 0.1476$$

TESTS OF SIGNIFICANCE

$$\hat{p} = \frac{3}{15}$$

Uses/Purposes:

1. What are these tests used for?

- To assess...

- evidence for/against a claim from a sample.
- evidence = prob.

2. What do these tests compare?

Samples/data
to hypothesis (claim)

- testing a claim
- given a claim
- take a sample
- decide whether claim is T/F

0.55

claim = 0.5

$$\frac{f}{15}$$

3. How are the results of these tests expressed?

probability of how well data
agrees w/ hypothesis

4. What are the results a probability of?

getting our sample (or something
more extreme) if claim is true

5. What are the ~~2~~ components of a Test of Significance?

a. Assumptions (state + check)

b. Hypotheses

c. Test Statistic

d. P-value (probability)

e. Conclusion

In Ch. 8, we will be looking at testing a population proportion (\hat{p})

Ch. 7 - μ

Components:

1. Hypotheses

What do the hypotheses always describe?

population

What types of symbols do the hypotheses always use?

parameters

Name: Null Hypothesis

Symbol:

H_0 :

What is it?

- claimed value
- known / believed value

FORM:

H_0 : parameter = value

Example:

$H_0: p = 0.5$

If the null hypothesis is true, what do we expect from the estimate?

estimate/sample \approx parameter

Name: Alternative Hypothesis

Symbol: H_a :

What is it?

- what we/you believe is true
- what you're hoping to find evidence for

FORM:

H_a : parameter $\begin{matrix} \geq \\ \leq \\ \neq \end{matrix}$ value

Two types of alternative hypotheses:

one sided: $>$, $<$

two sided: \neq

Examples:

$$H_a: p < 0.5$$

$$H_a: \mu > 65$$

$$H_a: p \neq 0.25$$

2. Tests Statistic

What does the test statistic measure?

compatibility b tw H_0 (claim) + data

What is this test statistic used for?

prob. calculation

What type of a variable is this test statistic and what type of distribution does it have?

Z-score $N(0,1)$

Formula (for the test statistic of a population proportion):

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Example:

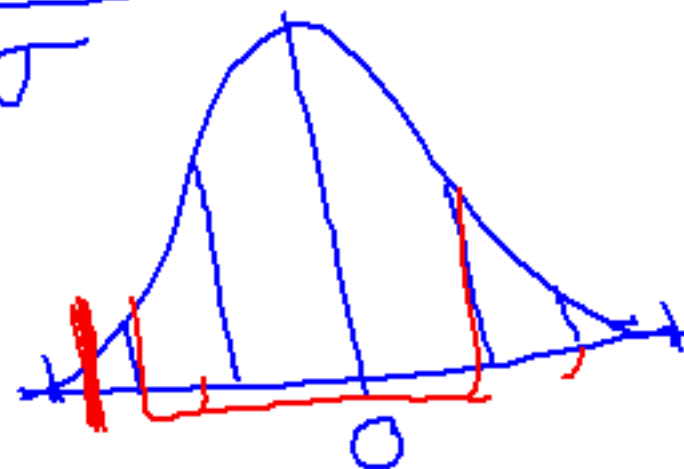
$$\hat{p} = \frac{2}{15}$$

$$p = 0.5$$

$$Z = \frac{0.133 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{15}}}$$

$$= -2.84$$

$$Z = \frac{x - \mu}{\sigma}$$



3. P-Value (prob)

Definition:

prob. of getting our sample or something more extreme) if claim H_0 is true

The smaller the p-value...

the more evidence against claim H_0

What is true about our evidence as the sample proportion (\hat{p}) gets further from the population proportion (p)?

evidence gets stronger against H_0 .

How do we use the test statistic to find the P-Value? What distribution do we use?

use normalcdf $N(0,1)$

$$P(Z \geq \text{test stat})$$

Ex: $P(Z < -2.84) = \text{normalcdf}(-\infty, -2.84, 0, 1) = 0.0022$

Statistical Significance

Definition:

result that happens so often (or so rarely) that it's not just chance.

What does statistical significance compare?

p-value to a fixed level (significance level)

What is a significance level? What symbol do we use to denote this level?

 α

$\alpha = \text{alpha} = \text{significance level}$

What does a significance level of 0.05 or 0.01 indicate about the null hypothesis?

$\alpha = \begin{matrix} 0.01 \\ 0.05 \\ 0.10 \end{matrix} \left. \vphantom{\begin{matrix} 0.01 \\ 0.05 \\ 0.10 \end{matrix}} \right\} \text{given}$

If a significance level is not given, what level do we use?

$\alpha = 0.05$

fixed value
when we
believe/don't
believe claim

Conclusions: = 2 sentences

What are the two conclusions that we can have (be careful how they are stated!)

$H_0: p = \text{---}$
 $H_a: p > \text{---}$
 \neq

- We reject H_0 in favor of H_a , b/c p-value $< \alpha$.
We have sufficient evidence that
the prop of (context) is \neq value.
 - We fail to reject H_0 b/c p-value $> \alpha$.
(NOT accept)
We have suff. evid. that the prop
of context is = to value.
* innocent vs. not guilty
- Our conclusion must always be a complete sentences in terms of H_0 .

Tests of Significance Example:

Go back to the alcohol abuse example. The National Board of Statistics claims that over 15% of college students are classified as binge drinkers. Remember, there was an SRS of 17096 college students and found 3314 were classified as binge drinkers. Is there evidence to support the Board of Statistic's claim? Use $\alpha = 0.05$.

1. Hypotheses:

$$H_0: p = 0.15$$

$$H_a: p > 0.15$$

$$n = 17096$$

$$X = 3314$$

$$\alpha = 0.05$$

STATE

$$\textcircled{1} \text{ SRS}$$

$$\textcircled{2} np, n(1-p) \geq 10$$

$$\textcircled{3} pop \geq 10 \cdot n$$

CHECK

$$\textcircled{1} \text{ circled}$$

$$\textcircled{2} \frac{(17096)(0.15)}{(17096)(0.85)} \neq 10$$

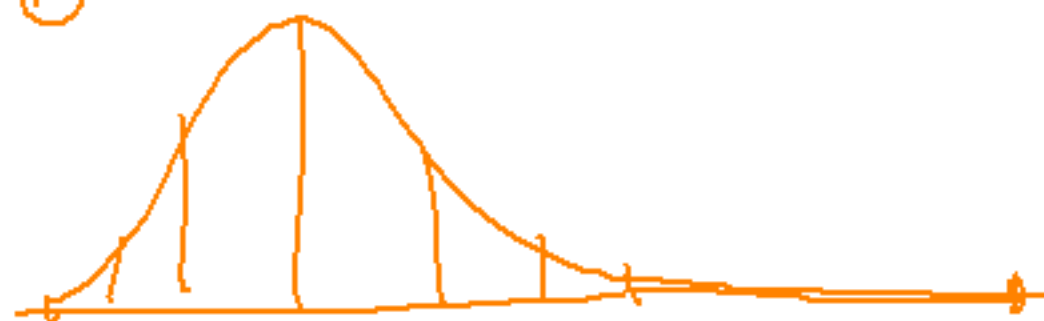
$$\textcircled{3} pop \neq 10(17096)$$

2. Test Statistic:

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{\left(\frac{3314}{17096}\right) - 0.15}{\sqrt{\frac{(0.15)(0.85)}{17096}}} = 16.056$$

3. P-Value:

$$P(Z > 16.056) = 2.756 \times 10^{-58}$$



4. Conclusion:

We reject H_0 in favor of H_a b/c p-val $< \alpha = 0.05$.
We have suff. evid. that prop. of binge drinkers is greater than 15%.

The proportion of people who are afraid of flying is claimed by the FAA to only be 35%. You do not believe this, and take a sample of 145 random adults and find that 70 of them are afraid of flying. Perform a statistical test of significance on the claim at the 0.05 significance level.

- (a) State the hypotheses

$$H_0: p = 0.35$$

$$H_a: p \neq 0.35$$

- (b) Find the test statistic

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = 3.352$$

- (c) Find the P-value, and sketch a picture of the P-value on a standard normal curve

$$2 \cdot P(Z > 3.352) \approx 8.035 \times 10^{-4}$$

$$2 \cdot P(Z < -3.352)$$

- (d) Report your conclusion at the $\alpha = 0.05$ level.

- reject H_0

- suff. evid $p \neq 0.35$

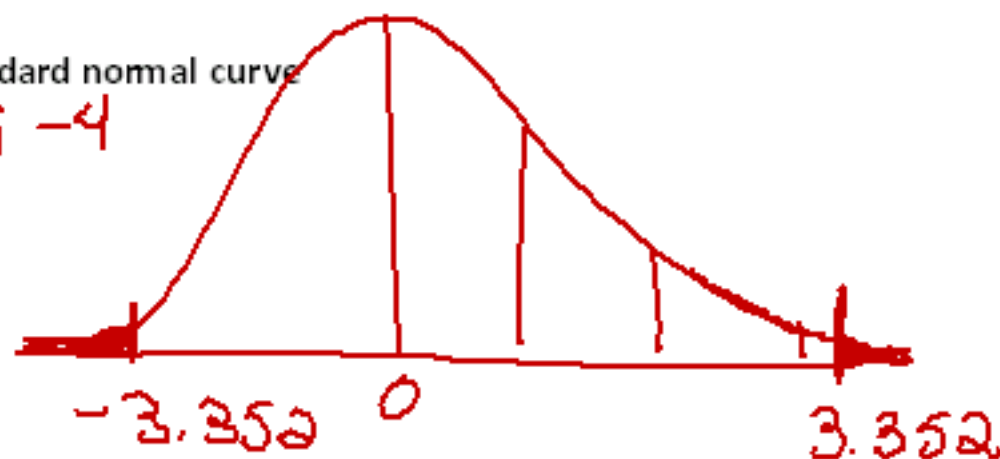
Assump

$$n = 145$$

$$x = 70$$

$$\alpha = 0.05$$

$$\hat{p} = \frac{70}{145}$$



- (e) Report your conclusion at the $\alpha = 0.01$ level.

$$p\text{-val} = 0.02$$

— Same

$$b/c \ p\text{-val} < 0.01$$

- (f) Create a 95% confidence interval for the population proportion and interpret

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (0.4014, 0.5641)$$

We are 95% conf. that the prop.
of people afraid to fly is btw.
40.14% & 56.41%

Complete the following worksheet
Section 8.1 - Hypothesis Testing

$\hat{p} = \frac{14}{40}$
 $p = 0.384$
 $X = 25$
 $n = 40$

- ② ASSUMP STATE
- ① SKS
 - ② np
 $n(1-p) \geq 10$
 - ③ $pop \geq 10 \cdot n$

- CHECK
- ① assumed
 - ② $(40)(0.384) \geq 10$
 $(40)(1-0.384) \geq 10$
 - ③ $pop \geq (10)(40)$

a) $H_0: p = 0.384$
 $H_a: p > 0.384$

b) $z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{(\frac{25}{40}) - 0.384}{\sqrt{\frac{(0.384)(1-0.384)}{40}}} = 3.134$
 p-value small

$P(z > 3.134) = 8.623 \times 10^{-4}$

We reject H_0 in favor of H_a b/c p-value $< \alpha = 0.05$.
 We have suff. evid that the prop. of free throws
 made is greater than 0.384.

$$d) \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \text{no } H_0: p =$$

$$= (0.5, 0.75)$$

$$x = 25$$

$$n = 40$$

$$C = 90\%$$

$$= 0.90$$

$$95\%$$

We are 90% conf. that
the prop of free throws made
is btw 50% & 75%

Calc/Form Sheet

Conf. Int: CALC: STAT \rightarrow TESTS \rightarrow 1 Prop Z-Int

Form sheet: $\text{statistic} \pm \left(\begin{smallmatrix} \text{crit} \\ \text{val} \end{smallmatrix} \right) \left(\begin{smallmatrix} \text{std. dev} \\ \text{of stat} \end{smallmatrix} \right)$

\swarrow tables below

Tests: Form Sheet:

$\frac{\text{statistic} - \text{param}}{\left(\begin{smallmatrix} \text{std. dev} \\ \text{of stat} \end{smallmatrix} \right)}$

calc:

$$\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$